Multi-layer Flight Control Synthesis and Analysis of a Small-scale UAV Helicopter

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Abstract—In this paper, we present a systematic procedure for the design of a controller for an autonomous helicopter. The studied model of the helicopter is a semi-linearized model, which includes some nonlinear parts that are separated from the linear blocks. The controller structure is a hierarchical controller which consists of two layers: the inner-loop controller that covers the linear parts of the model and the outer-loop controller that handles the nonlinear parts. The inner-loop controller aims at the attitude control of the helicopter and the outer-loop is responsible for its position control. The performance of the designed controller is demonstrated through the simulation and actual flight tests in the hovering situation and the path-tracking mode.

Index Terms—Unmanned Aerial Vehicles, Hierarchical Control, Inner-loop Controller, Outer-loop Controller.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have attracted the attention from the academic and military communities due to their various capabilities and their potential to address numerous applications in civilian and military areas (see e.g., [1], [2], [3], [4]). An essential issue in the aerial robotics is designing a reliable controller for the UAVs to make them able to perform flight missions autonomously. However, due to the complex structure of the UAVs, their controller design is not an easy task and has motivated several research groups to focus on the design and improvement of their UAV controller performance [1], [5], [6]. In this paper, we propose a systematic controller design procedure for the UAV system. Our test bed is a helicopter which is developed by our research group in the National University of Singapore [7], [8]. The proposed controller has a hierarchical structure, which consists of two layers: The inner-loop controller and The outer-loop controller.

The inner-loop is responsible for the control of the attitude of the UAV and the outer-loop is used to drive the UAV into the desired position. The model of this UAV is a semi-linearized model, in which the nonlinear terms are separated from the linear parts and are brought into the outer-loop. Therefore, the inner-loop includes only the linear parts and we use the $H_{\infty}$ control design technique to achieve the desired performance and to reduce the disturbance effect. For the outer-loop, we first compensate for the nonlinear terms, and then, we use a proportional controller (P-controller) that enables the system to follow the desired path.

The rest of this paper is organized as follows. In Section II, the structure and the model of the NUS UAV are described. It is a decoupled system which includes two independent subsystems. The controller structure for this model of UAV is explained in Section III. The actual flight tests are conducted to evaluate the controller performance. The experimental results are demonstrated in Section IV. Finally, the paper is concluded in Section V.

II. SEMI-LINEARIZED MODEL OF THE UAV

To construct a UAV helicopter (Fig. 1), one can buy a commercial radio-control helicopter and upgrade it to be able to perform an automatic flight. To develop such an autonomous vehicle, it is necessary to design a reliable avionic system including the following parts:

- Onboard computer system and its extension boards that are responsible for receiving data from the sensors, processing the collected data, and generating the control signals.
- The sensors that are responsible for measuring the state variables such as velocity, acceleration, fuel level, attitude, and etc.
- The servos that are used to manipulate the angles of the helicopter blades to move the UAV towards the desired direction.

In [9], the procedure of the construction of such a UAV helicopter is described, and in [7], its hardware parts including both the avionic system and the ground station are illustrated in detail.

The model of the UAV can be obtained based on basic physical principles. However, the achieved model might be a nonlinear model [8]. As a rational way, we have linearized the UAV model at the hovering state, in which the linear and angular velocities, the pitch angle, and the roll angle are kept close to zero. The resulting semi-linearized model of the NUS
UAV has been presented in [6]. The model consists of two decoupled subsystems as follows:

\[
\begin{align*}
\dot{x}_1 &= A_1 x_1 + B_1 u_1 \\
\dot{x}_{out1} &= C_1 x_1
\end{align*}
\]  

(1)

and

\[
\begin{align*}
\dot{x}_2 &= A_2 x_2 + B_2 u_2 \\
\dot{x}_{out2} &= R'C_2 x_2
\end{align*}
\]  

(2)

where \( x_1 = [ V_x(m/s) \omega_x(rad/s) \omega_z(rad/s) \omega_y(rad/s) \phi(rad) \theta(rad) \delta_{out}(rad) ]' \) and \( x_2 = [ \dot{\delta}_{pitch}(rad) \dot{\delta}_{roll}(rad) ]' \) are the inner-loop state variables; \( x_{out1} = [ z(m) \psi(rad) ]' \) and \( x_{out2} = [ x(m) y(m) ]' \) are the outer-loop state variables; \( u_1 = [ \delta_{col}(rad) \delta_{pedal}(rad) ]' \) and \( u_2 = [ \delta_{roll}(rad) \delta_{pitch}(rad) ]' \) are the control inputs. Here, \( V_x, V_y, \) and \( V_z \) are the linear velocities; \( \omega_x, \omega_y, \) and \( \omega_z \) are the angular velocities; \( \phi, \theta, \) and \( \psi \) are the Euler angles; \( \delta_{col} \) and \( \delta_{pedal} \) are the cyclic commands, and \( \delta_{pedal} \) is the pedal channel input that affects the yaw angle, and \( \delta_{col} \) is the collective channel. The saturation level for all of the control inputs is ±0.5.

Matrices \( A_1, A_2, B_1, B_2, C_1, C_2, \) and \( R \) are as follows:

\[
A_1 = \begin{bmatrix}
-0.6821 & -0.1070 & 0 \\
-0.1446 & -5.5561 & -36.6740 \\
0 & 2.7492 & -11.1120
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
15.6491 & 0 \\
1.6349 & -58.4053 \\
0 & 0
\end{bmatrix},
\]

\[
C_1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
-0.1778 & 0 & 0 & 0 & -9.7807 \\
0 & -0.3104 & 0 & 0 & 9.7807 \\
-0.3326 & -0.5353 & 0 & 0 & 0 \\
0.1903 & -0.2940 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
B_2 = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0.0496 & 2.6224 \\
2.4928 & 0.1740
\end{bmatrix},
\]

\[
C_2 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

and

\[
R = \begin{bmatrix}
\cos \psi & -\sin \psi \\
\sin \psi & \cos \psi
\end{bmatrix}.
\]

The model structure of the UAV is shown in Fig. 2. Subsystem 1 and Subsystem 2 are similar; however, in Subsystem 2, there exists the term \( R' \) as a nonlinear term that makes the design procedure different. In the following section, the design control procedures for this model of UAV is discussed in detail.

\[
\begin{align*}
\delta_x &= A_1 x_1 + B_1 u_1 \\
\dot{x}_1 &= A_2 x_2 + B_2 u_2 \\
\dot{x}_{out1} &= R'C_2 x_2
\end{align*}
\]  

(3)

III. CONTROLLER DESIGN

We design the flight controller in two steps. First, we control the attitude of the UAV in the inner-loop, and then, we will control the position of the UAV and drive it towards the desired position in the outer-loop. The inner-loops of both subsystems are linear and therefore, we can use the \( H_{\infty} \) control design technique to stabilize the system and achieve the desired performance. Also the designed controller will reduce the
disturbance effect on the control performance. For the outer-
loop, we use a P-controller to track the position references.
The design procedure for both subsystems is similar. However,
Subsystem 2 has the nonlinear term $R'$, in its outer-loop
that makes the design procedure more difficult. We need to
first compensate for this nonlinearity, and then, design a P-
controller for the system to achieve the required performance.
In the following parts, we will describe the controller design
procedure for the inner-loop and the outer-loop of the system.

A. Inner-loop Controller Design

The inner-loop controllers in both subsystems aim at
the control of the attitude of the UAV. The attitude of the
UAV is captured by the state variables $x_1$ and $x_2$ and their
 corresponding linear dynamics, as described in (1) and (2).
For these linear multi-variable systems, we can use any of
the classical control tools. However, with the $H_\infty$ control
design technique, both the robust stability and the desired
performance of the system are achievable [11]. Utilizing the
$H_\infty$ control design technique, we have designed the control
law in the form of $u_i = F_i x_i + G_i r_i$, $i = 1, 2$, where $F_i$
can be obtained from the $H_\infty$ design technique, and $G_i$,
$i = 1, 2$, are the feedforward gains that can be calculated
as $G_i = -(C_i(A_i + B_i F_i)^{-1} B_i)^{-1}$. Using this method, the
matrices $F_1, G_1, F_2,$ and $G_2$ are as follows:

$$F_1 = \begin{bmatrix}
-0.0035 & -0.0005 & 0.0027 \\
0.0008 & 0.0364 & -0.0481 \\
\end{bmatrix}$$

$$G_1 = \begin{bmatrix}
0.1371 & 0.0066 \\
-0.0020 & -0.2748 \\
\end{bmatrix}$$

$$F_2 = \begin{bmatrix}
0.0017 & -0.1683 & -0.0486 & 0.0081 \\
0.0815 & -0.0461 & -0.0087 & -0.0535 \\
-1.9336 & -0.1974 & -0.3227 & -2.1444 \\
-0.3908 & -1.0690 & -1.1712 & -0.4659 \\
\end{bmatrix}$$

and

$$G_2 = -(C_2(A_2 + B_2 F_2)^{-1})^{-1} = \begin{bmatrix}
-0.0029 & 0.2335 \\
-0.0978 & 0.0632 \\
\end{bmatrix}$$

To simulate the resulting controlled system and evaluate the
controller performance, we ran a simulation (Fig. 3) with
the initial state of $x_1(0) = [1 0 0]'$ and $x_2(0) = [1 0 0 0 0.25 0 0 0]'$. The controlled system reaches
the steady hovering state after 3 second, while the control inputs
are within the unsaturated region.

B. Outer-loop Controller Design

In the outer-loop of Subsystem 1, we have used a P-
controller to drive the UAV to follow the desired path. Before
designing the P-controller, we have compensated for the
nonlinear term $R'$ in the outer-loop of Subsystem 2. The
nonlinearity introduced by $R'$, can be compensated by the term
$RR'$, as it is a transformation matrix and it has the property that
$RR' = I$. To explain the elimination of the nonlinear terms in
Subsystem 2, the designed control structure is shown in Fig. 4,
where $G_{in2} = C_2(SI - (A_2 + B_2 F_2)^{-1} B_2 G_2$ is the inner-
loop block. Figure 5 shows that the block $G_{in2}$ is very close to
a decoupled system with equal diagonal elements. Using this
property, we can exchange the blocks $G_{in2}$ and $R'$ so that the
rotation matrices $R$ and $R'$ will cancel each other.

![Fig. 4. The control structure of Subsystem 2.](image1)

![Fig. 5. Bode plot of the entries of $G_{in2}$.](image2)

After the cancelation of the nonlinear terms, we can design
the proportional control gains $K_{p1}$ and $K_{p2}$ using generalized
Nyquist theorem [12]. The control gains $K_{p1}$ and $K_{p2}$ were
obtained as follows:

$$K_{p2} = diag\{0.3, 0.3\}, \quad K_{p1} = diag\{0.5, 0.7\}$$

In these control parameters, the diagonal elements of $K_{p2}$
are selected equal, due to the fact that the behavior of the UAV
system in the $x$ and $y$ directions should be the same, but for
$K_{p1}$ we do not have this kind of restriction. The whole control
structure is shown in Fig. 6.

![Fig. 6. Multi-layer control structure of the UAV.](image3)

To simulate the resulting system, let the outer-loop reference
at $r_1 = (1, 0.5)$ and $r_2 = (1, 1)$, and consider that the current
position of the UAV is located at $x_{out1u} = (0, 0)$ and $x_{out2u} =
(0, 0). The system will smoothly reach its target after around 8 second, (Fig. 7).

IV. EXPERIMENTAL RESULTS

We used the designed controller in an actual flight test to verify the simulation results. First, we used the UAV in a hovering situation at \((x, y, z, \Psi) = (-30, -10.3, 8, -1.3)\) for 15 sec, as shown in Fig. 8. The result is quite impressive, as during this period, the UAV deviates from the desired point at most 1 meter (Fig. 9). The control inputs are shown in Fig. 10. All the control inputs are within the unsaturated region.
Next, we have driven the UAV to track a zigzag path as a given trajectory. In this experiment the altitude and the heading angle of the UAV remain unchanged (Fig. 11). To evaluate the controller performance, both the reference path and the UAV position are shown in Fig. 12.

The small errors in these two experiments can be because of the environmental effects such as wind disturbance and the inaccuracy of the GPS sensor. The results show that the UAV is able to track the desired trajectory as long as the generated path does not push the UAV far from the hovering state. The videos of the hovering and path tracking experiments are available at the web links, http://uav.ece.nus.edu.sg/video/hover.mpg and http://uav.ece.nus.edu.sg/video/zigzag.mpg, respectively.

V. CONCLUSION

In this paper, we proposed a hierarchical controller for the NUS UAV helicopter. The lower level is an $H_{\infty}$ controller, which is responsible for the control of the attitude of the UAV. It provided a stable and robust control performance for the UAV in the hovering mode. The higher level aims at the position control of the UAV and consists of a P-controller combined with a nonlinear transformation. This
control structure makes the UAV able to perform the path tracking mission. The experimental results show that the UAV is able to follow the generated path as long as it respects the physical constraints on the UAV motion. The controlled system was evaluated through the hovering and path tracking experiments. The simulation results and actual flight tests show the effectiveness of the designed controller.

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